The Fifth International Bridge Seismic Workshop (5IBSW)
Hunan University, Changsha, China, June 5-7, 2026

SIMPLIFIED DISPLACEMENT-BASED ANALYSIS OF ISOLATED BRIDGES ON YIELDING PIERS

I. G. Buckle [(1)]

[(1)] *Foundation Professor Emeritus, University of Nevada Reno, United States of America, igbuckle@unr.edu*

Abstract

Most seismically isolated bridges are designed to keep the columns elastic during the design earthquake. Since an isolated bridge on elastic columns is essentially a single-degree-of-freedom system, a simplified direct-displacement method of analysis is widely used for their design that iterates on the superstructure displacement. Multimode Response Spectrum Analysis and/or Response History Analysis are not necessary unless the bridge has complex geometry. However, performance in an earthquake larger than the design earthquake is not explicitly considered except some codes require the isolator to have capacity (Δ_C) for at least 1.5 times the displacement during the design earthquake (Δ_{DE}). Whether the column is still elastic at this displacement is not explicitly checked and if the demand is greater than Δ_C and the isolator runs into a hard stop or becomes unstable, extreme demands may be placed on the column. This paper extends the direct-displacement method of analysis for bridges with elastic columns to those with yielding columns in order to give insight into their behavior under larger-than-design ground motions. An example is given to illustrate the methodology using a simple spreadsheet. Finally, it is noted the methodology may also be used to study the economic benefits of allowing column yield during design ground motion, but this option is not explored in this paper.

Keywords*: seismically isolated bridge; larger-than-design earthquake; yielding columns; direct displacement-based analysis; spreadsheet solution*

1. Introduction

Most seismically isolated bridges are designed to keep the columns elastic during the design earthquake. Since an isolated bridge on elastic columns is essentially a single-degree-of-freedom system, a simplified direct-displacement method of analysis is widely used for their design that iterates on the superstructure displacement. Multimode Response Spectrum Analysis and/or Response History Analysis are not necessary unless the bridge has complex geometry. However, performance in an earthquake larger than the design earthquake is not explicitly considered except some codes require the isolator have capacity (Δ_C) for at least 1.5 times the displacement during the design earthquake (Δ_{DE}). Whether the column is still elastic at this displacement is not explicitly checked and if the demand is greater than Δ_C and the isolator runs into a hard stop or becomes unstable, extreme demands may be placed on the column. This paper extends the direct-displacement method of analysis for bridges with elastic columns that is described in AASHTO [1] to those with yielding columns, in order to give insight into their behavior under larger-than-design ground motions.

2. Approach

When the substructures below the isolators can yield (columns and abutment piles), the isolators will have different displacements because the substructures will, in general, have different effective stiffnesses due to variations in column height, yield moment, and drift in each column. To apply the simplified displacement method in this situation, the effective stiffness of the bridge, K_{eff}, must be modified to include the secant stiffness of the yielding substructure as well as the secant stiffness of the yielding isolator. Since both stiffnesses are displacement dependent, the solution is iterative. Once K_{eff} is known, the method follows the same steps, and uses the same basic equations, as for an isolated bridge with non-yielding substructures.

3. Basic Equations

3.1 Effective stiffness of bridge with yielding substructures, K_{eff}

The effective stiffness of an isolated superstructure on a yielding substructure is obtained by summing the effective stiffnesses of the combined isolator-column substructures. Fig. 1 shows the deformations in an idealized pier, comprising a superstructure, isolator, and yielding column. Both the isolator and column are assumed to have bilinear properties, as shown. It is noted that the overall response of the combined isolator /substructure system is given by a trilinear hysteretic curve as shown.

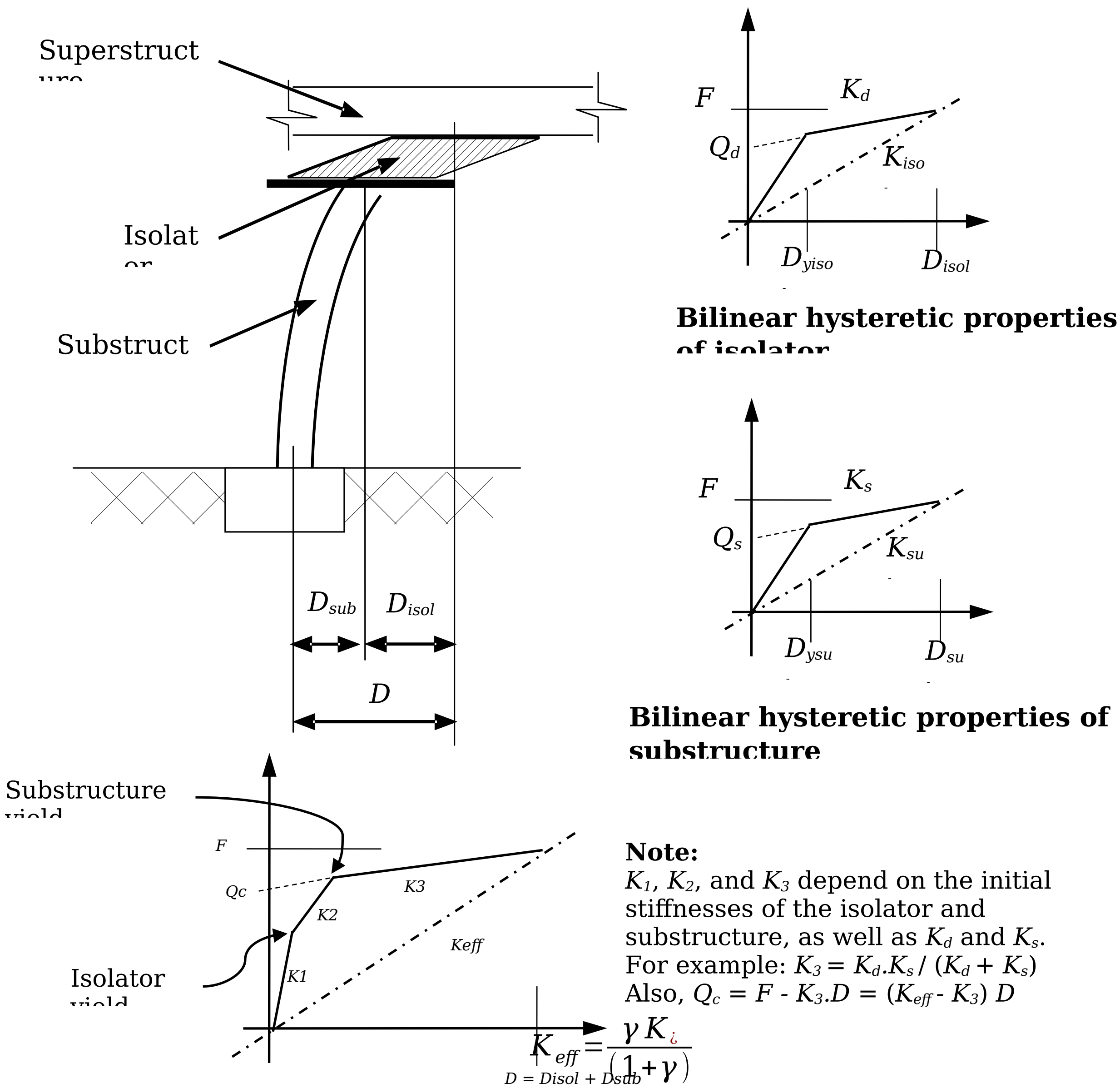

Fig. 1 - Trilinear hysteretic properties of bridge with yielding isolators on yielding substructures

$$K_{eff}=\frac{\gamma K_{¿}}{(1+\gamma)} \tag{1}$$

where

$$\gamma=\frac{K_{isol}}{K_{¿}}=\frac{(K_d D+Q_d-Q_s)}{(K_{¿} D-Q_d+Q_s)} \tag{2}$$

$$K_{isol}=effective\ stiffness\ of isolator\ at\ displacement\ D_{isol}=\frac{Q_d}{D_{isol}}+K_d \tag{3}$$

$$K_{¿}=effective\ stiffness\ of\ substructure\ at\ displacement\ D_{¿}=\frac{Q_s}{D_{¿}}+K_s \tag{4}$$

$$D_{isol} = isolator\ displacement = \frac{D}{(1+\gamma)} \quad (5)$$

$$D = displacement\ of\ superstructure = D_{¿} + D_{isol} \quad (6)$$

and $$D_{¿} = substructure\ displacement = column\ drift \quad (7)$$

Also $$D_{yisol} = \frac{Q_d}{(K_{id} - K_d)} \wedge D_{ysub} = \frac{Q_s}{(K_{is} - K_s)} \quad (8)$$

$$where\ K_{id} \wedge K_{is}\ are\ initial\ stiffnesses\ of\ the\ isolator \wedge substructure\ res$$

The effective stiffness of substructure j, $K_{eff,j}$, can be shown to be given by Eq. (1), where the subscript 'j' is omitted for clarity, as well as in Eq. (2) - (8).

It follows that the effective stiffness, K_{eff}, for the complete bridge with N substructures in parallel with each other, is given by:

$$K_{eff} = \sum_{j=1}^{N} K_{eff,j} \quad (9)$$

3.2 Effective damping ratio, ζ_{eff}

The damping ratio includes the flexibility of the substructure and for substructure j is given by:

$$\zeta_{eff,j} = \frac{2}{\pi} \frac{Q_{d,j}}{F_j} \left(\frac{D_{isol,j} - D_{yisol,j}}{D} \right) \quad (10a)$$

where $F_j = K_{eff,j} . D$.
The damping ratio, ζ_{eff}, for the complete bridge is not given by simply adding the ratios for the individual substructures but by:

$$\zeta_{eff} = \frac{2}{\pi} \frac{\sum_{1}^{N} Q_{d,j}(D_{isol,j} - D_{yisol,j})}{FD} = \frac{2}{\pi} \frac{\sum_{1}^{N} Q_{d,j}(D_{isol,j} - D_{yisol,j})}{K_{eff} D^2} \quad (10b)$$

3.3 Substructure and isolator forces F_{sub} and F_{isol}

The force in any substructure is given by:

$$F_{¿} = K_{¿} D_{¿} \quad (11)$$

where, from Eq. (6), D_{sub} = substructure displacement = D - D_{isol}

The force in the isolators supported by the substructure is given by:

$$F_{isol} = K_{isol} D_{isol} \quad (12)$$

It is noted that these two forces should equal one another, since the isolator and substructure 'see' the same shear force due to the serial nature of the load path.

4. Basic Steps

As noted above, the solution is iterative since many of the key parameters describing the properties of the bridge (K_{eff}, T_{eff}, and ζ_{eff}) depend on the displacement of the bridge, D, which is not known at the beginning of the analysis. The method therefore begins by assuming a displacement for the bridge superstructure, D, and iterating until convergence is achieved, usually within a few cycles. The steps are as follows:

1. Assume a value for the superstructure displacement D. To help with this estimate, it is suggested to use the Displacement Response Spectrum for the design ground motion, assuming an isolated period (T) of 1.5 sec and 5% damping. It is noted that if the Acceleration Response Spectrum is inversely proportional to period in the isolated-period range, the Displacement Response Spectrum is given by

$$D = \frac{g}{4\pi^2} \frac{S_{D1}}{B_L} T \tag{13}$$

   where S_{D1} is the spectral acceleration coefficient at 1.0 sec, including site effects, and B_L=1 for 5% damping.
2. Calculate the effective stiffness of the bridge, K_{eff}, from Eq. (1).
3. Calculate the effective period, T_{eff}, from:

$$T_{eff} = 2\pi \sqrt{\frac{W}{g K_{eff}}} \tag{14}$$

   where W is the weight of the superstructure.
4. Calculate the equivalent viscous damping ratio for complete structure, ζ_{eff}, from Eq. (10b).
5. Obtain damping factor, B_L, from:

$$B_L = \left(\frac{\zeta_{eff}}{0.05}\right)^{0.3} \tag{15}$$

6. Calculate displacement, D, from Eq. (13).
7. Compare calculated value for displacement, D, with that assumed in Step 1. If in close agreement, go to Step 8; if not, repeat from Step 2 using the value for D from Step 6.
8. Calculate the total base shear in the structure, F, from K_{eff} D or by summing the substructure forces given by Eq. (11). Isolator forces are equal to the substructure forces or may be found from Eq. (12). Isolator and substructure displacements are given by Eqs. (5) and (7) respectively.

5. Example

5.1 Bridge Description

The superstructure of a two-span bridge weighs 2,370 kN. It is located on a site where S_{D1} (spectral acceleration coefficient at 1.0 sec) is 0.55. The center pier is a single, 925 mm diameter, reinforced concrete column, 7.6 m high, fixed at the base and pinned at the top. The elastic modulus for the concrete is 30 GPa, and the effective column moment of inertia is 80% of gross inertia. The lateral stiffness of each abutment is 1,750 kN/mm.

The bridge is isolated with bearings at the abutments and over the pier. Total values for Q_d and K_d summed over all the isolators are 0.075 W (= 177.75 kN) and 2.275 kN/mm respectively. Isolator properties and the weight carried at each substructure are given in Table 1.

Substructure	**Weight carried (kN)**	**Qd (kN)**	**Kd (kN/mm)**
North Abutment	445	33.375	0.427
Pier	1,480	111.00	1.421
South Abutment	445	33.375	0.427
Totals	**2,370**	**177.75**	**2.275**
Note: Isolators are assumed to have negligible yield displacements (i.e., $D_{yisol} << D_{isol}$).			

The Q_s value for the column is found from a section analysis and taken to be 200 kN in this example to ensure the isolator yields before the column ($Q_s > Q_d$). The Ks value is taken as one-tenth of the elastic stiffness of the column assuming a cracked section.

Table 1- Isolator properties

Using the methodology in the previous section, the displacement of the superstructure, the total base shear, and the distribution of this shear to the three substructures (two abutments and pier) is found for the given earthquake ground motion (S_{D1}= 0.55).

5.2 Solution

An excel spreadsheet is constructed to obtain the solution as shown in Table 2.

A key step is the calculation of the effective stiffness for the complete bridge system (Step 2) which requires the summation of the effective stiffnesses of each substructure. This is shown in a separate section of the spreadsheet, which is entered with an estimate of the displacement (183 mm). The effective stiffness is then calculated at that displacement (2.582 kN/mm). This value is then used in

Step 3 to calculate the effective period and the remaining steps follow. The solution shown in the table is the final trial after convergence has been obtained. Intermediate trials are not shown. It will be seen that the section of the spreadsheet that calculates effective stiffness, also calculates the displacements in the isolators and substructures, and the shears in the substructures, the both in absolute terms and as a percentage of the total base shear.

Results are summarized as follows:

Total displacement superstructure = 183 mm
Total base shear = 474 kN
(20.0% W)

Substructure shear forces:

- North abutment shear = 112 kN (23.6% of total base shear)
- Pier shear = 250 kN (52.8% of total base shear)
- South abutment shear = 112 kN (23.6% of total base shear)

Isolator and substructure displacements:

- at north abutment: 183 mm (isolator), 0 mm (abutment)
- at pier: 98 mm (isolator), 85 mm (pier)
- at south abutment: 183 mm (isolator), 0 mm (abutment)

SIMPLIFIED DISPLACEMENT-BASED METHOD: 2-span isolated bridge with yielding pier: W= 2,370 kN, S_{D1}=0.55

YIELDING PIER
Based on CEE 729 Bridge Example with Elastic Pier

BRIDGE AND SITE PROPERTIES

Acceleration coefficient, S_{D1}	0.55			Ecol	30.00 GPa	Qsabut	0.00	kN			
Superstructure weight, W	2370.00	g/4pi^2	248.49 mm/s^2	diam	0.925 m	Ksabut	1750.00	kN/mm			
Qd / W	0.075	f	250.00	Icol	0.035937 m^4	Qscol	200.00	kN	Dyisol	0.00	mm
				hcol	7.60 m	Kscol	0.589	kN/mm	Dysub	37.70	mm
				Kcol	5.894 kN/mm	(0.8x3EI/h^3, cracked section)					

BRIDGE RESPONSE FOR Qd = 0.075W and Kd = 2.275 kN/mm **D** 183.50

Step	Parameter	Trial	Substructure	Wsub	Qd	Kd	Qs	Ks	γ	Ksub	Keff	Disol	Dsub	Qd(di-dy)	Fisol	Fsub	Fsub %
	Characteristic strength, Q_d	177.75	N. Abut	445.00	33.38	0.427	0.00	1750.00	0.0003	1750.00	0.609	183.44	0.06	6122.182	111.70	111.70	23.58%
	Post-yield stiffness, K_d	2.275	Pier	1,480.00	111.00	1.421	200.00	0.589	0.8711	2.930	1.364	98.07	85.43	10885.610	250.36	250.36	52.84%
1	Assumed displacement, D	183.50	S. Abut	445.00	33.38	0.427	0.00	1750.00	0.0003	1750.00	0.609	183.44	0.06	6122.182	111.70	111.70	23.58%
2	Effective stiffness, K_{eff}	2.582	**Totals**	**2,370.00**	**177.75**	**2.275**					**2.582**			**23129.975**		**473.76**	**100.00%**
3	Effective period, T_{eff}	1.922															
4	Viscous damping ratio, ζ_{eff}	0.169										μ_{col}	2.27				
5	Damping factor, B_L	1.442															
6	Displacement, D (eq 2.29)	183.27															
8	Total base shear, F	473.16															
	F/W	19.96%															

NOTES

1. Results shown are for last cycle of iteration where convergence on displacement *D* has been reached.
2. Solution uses a displacement response spectrum that is linear with regard to period based on an acceleration spectrum that is inversely proportional to period. It can be modified for an 11- or 22-point or other acceleration spectrum as desired, using D(T) = A(T).T²/4π²
3. Solution assumes damping is dominated by isolator hysteresis and contribution from plastic deformation in column is negligible by comparisor
In this example the displacement ductility ratio for the column is 2.27, implying essentially elastic behavior and therefore low hysteretic dampi
An extension to include damping due to column hysteresis is in development.
4. For the purpose of demonstrating concept, this solution is coded in an excel spreadsheet and is for a yielding isolator on a yielding column. Spreadsheet can be manually adjusted for either an elastic isolator or elastic column or both. If parameter studies are envisaged involving wide range of component variables, a MatLab solution (or similar) might be preferable.

ielding pier

Table 2- Simplified displacement proposed analysis of 2-span isolated bridge with

Comparing these results with those obtained assuming an elastic column, the effect of yield in the pier is to increase the displacements by about 11% (from 165 to 183 mm) and reduce the total base shear by about 3% (from 487 to 474 kN). The main reason for this behavior is the lengthening of the effective period from 1.80 to 1.92 sec due to the increased flexibility of the columns as they yield and undergo plastic deformation.

6. Conclusions and Observations

The direct-displacement-based method of analysis for an isolated bridge with elastic columns [1] readily extends to isolated bridges with yielding columns, as shown in this paper. It may therefore be used to investigate bridge performance in earthquakes larger than the design ground motions when the columns are likely to yield. But it may also be used to study the economic benefits of allowing the columns to yield during design ground motions, which is permitted in the AASHTO Specifications [1] but with restrictions.

Some observations on the method presented above include:

- Solution uses a displacement response spectrum that is linear with regard to period based on an acceleration spectrum that is inversely proportional to period. It can be modified for an 11- or 22-point acceleration spectrum, or a site-specific spectrum, as desired using $D = A.T^2/4\pi^2$ where T is period at which displacement is to be calculated.
- Solution assumes damping is dominated by isolator hysteresis and contribution from plastic deformation in column is negligible by comparison. In this example the displacement ductility ratio for the column is 2.27, implying essentially elastic behavior and therefore low hysteretic damping. An extension to include damping due to column hysteresis is feasible.
- For the purpose of demonstrating the concept, this solution is coded in an excel spreadsheet and is for a yielding isolator on a yielding column. Spreadsheet can be manually adjusted for either (a) an elastic isolator on a yielding column, or (b) a yielding isolator on an elastic column. If parameter studies are envisaged involving a range of variables, a MatLab solution (or similar) might be preferable.

7. Reference

1. AASHTO (2010): *Guide Specifications for Seismic Isolation Design*, Third Edition, American Association State Highway Transportation Officials, Washington DC.