Time-domain dimension-reduction simulation for non-stationary stochastic ground motion

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Research Background

➢ Bridge infrastructures under stochastic dynamic actions

- Earthquakes, wind, and ocean waves are highly random in location and timing and may cause severe damage to civil engineering structures within a short period.
- Bridge failure lead to substantial casualties and economic losses, highlighting the need for accurate stochastic modeling.
- Measured records of disaster stochastic dynamic actions are scarce, while the demands for structural analysis, reliability assessment and uncertainty quantification continue to grow.
- Therefore, artificial simulation based on stochastic process modeling has become a commonly adopted approach.

Foundation for Reliability Assessment

Accurate modeling for stochastic dynamic actions

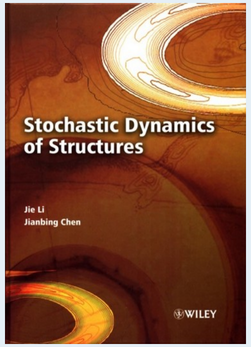
Theoretic Background

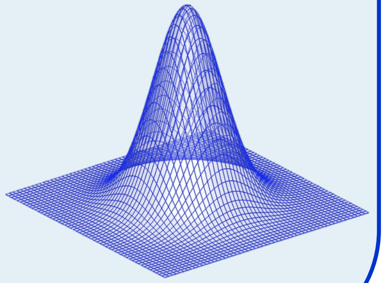
Transfer of randomness

Theoretical Basis

Probability Density Evolution Method

- PDEM is a core foundation of the third-generation structural design theory.
- Stochastic dynamic action modeling provides an important basis for engineering stochastic dynamics.



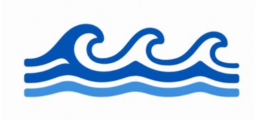


Input

Stocahastic dynamic actions

Earthquakes

Wind



Ocean waves

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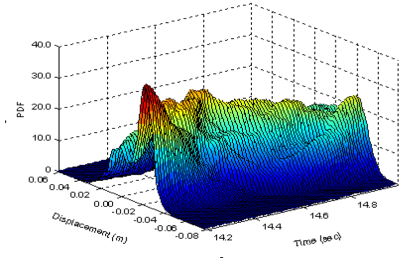
System

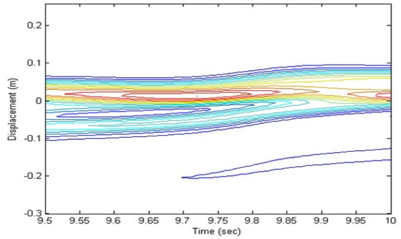
Bridge structures

Long-span bridges

Output

Stochastic dynamic





Probability distribution

Reliability assessment

Accurate stochastic action modeling is the basis for assessing the disaster-resilient reliability of bridge structures

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Theoretical Basis

Time-domain representations for stochastic processes

One-dimensional univariate stochastic process (1D-1V)

Fourier-Stieltjes integral form

$$\tilde{X}(t) = \int_{-\infty}^{\infty} A(t,\omega)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z(\omega) = \int_{-\infty}^{\infty} A(t,\omega)\mathrm{e}^{\mathrm{i}\omega t} H(\omega)\mathrm{d}U(\omega)$$

Time–frequency modulation function

Complex orthogonal increment processes

Its increments satisfy

$$E\left[\mathrm{d}Z(\omega)\right]=0,\quad E\left[\mathrm{d}U(\omega)\right]=0 \qquad \mathrm{d}Z(-\omega)=\mathrm{d}Z^*(\omega),\quad \mathrm{d}U(-\omega)=\mathrm{d}U^*(\omega)$$

$$E\left[\mathrm{d}U(\omega)\mathrm{d}U^*(\omega')\right]=\frac{1}{2\pi}\delta_{\omega\omega'}\mathrm{d}\omega \qquad E\left[\mathrm{d}Z(\omega)\mathrm{d}Z^*(\omega')\right]=\frac{1}{2\pi}S_X(\omega)\delta_{\omega\omega'}\mathrm{d}\omega$$

1 Stationary stochastic process

Auto-PSD

$$S_X(\omega)=\left|H(\omega)\right|^2$$

Frequency response function

Satisfying

$$H(-\omega)=H^*(\omega)$$

Fourier transform pairs

$$\begin{cases} H(\omega) = \int_{-\infty}^{\infty} h(\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau \\ h(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega \end{cases}$$

Time-domain representation

$$X(t) = \int_{-\infty}^{\infty} h(t-\tau)\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}U(\omega)\mathrm{d}\tau = \int_{-\infty}^{\infty} h(t-\tau)\mathrm{d}W(\tau) = \int_{-\infty}^{\infty} h(t-\tau)w(\tau)\mathrm{d}\tau$$

Where $w(\tau)$ **satisfies**

$$S_w(\omega)=1 \qquad E\left[\mathrm{d}w(\tau)\right]=0,\quad E\left[\mathrm{d}w(\tau)\mathrm{d}w(\tau')\right]=\delta(\tau-\tau')$$

2 Non-stationary stochastic process

EPSD

$$S_{\tilde{X}}(t,\omega)=\left|A(t,\omega)\right|^2 S_X(\omega)=\left|\tilde{H}(t,\omega)\right|^2$$

Fourier transform pairs

$$\tilde{H}(t,-\omega)=H^*(t,\omega)$$

$$\begin{cases} \tilde{H}(t,\omega) = \int_{-\infty}^{\infty} \tilde{h}(t,\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\tau \\ \tilde{h}(t,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{H}(t,\omega)\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}\omega \end{cases}$$

Time-domain representation

$$\tilde{\tilde{h}}(t,\tau)=h^*(t,\tau)$$

$$\begin{aligned}\tilde{X}(t) &= \int_{-\infty}^{\infty}\tilde{h}(t,t-\tau)\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}\omega\tau}\mathrm{d}U(\omega)\mathrm{d}\tau \\ &= \int_{-\infty}^{\infty}\tilde{h}(t,t-\tau)\mathrm{d}W(\tau) \\ &= \int_{-\infty}^{\infty}\tilde{h}(t,t-\tau)w(\tau)\mathrm{d}\tau\end{aligned}$$

Where $\mathrm{d}W(\tau)$ **satisfies**

$$E\left[\mathrm{d}W(\tau)\right]=0,\quad E\left[\mathrm{d}W(\tau)\mathrm{d}W^*(\tau')\right]=\delta_{\tau\tau'}\mathrm{d}\tau$$

★ 1. Stochastic processes can be regarded as the result of linear transformations driven by linear time-varying or time-invariant systems.

★ 2. Time-domain analysis methods are as crucial as frequency-domain approaches. Their discretizations give rise to two typical simulation methods: the SRM in the frequency domain and the linear filter method in time domain.

Theoretical Basis

1 Piecewise integral

$$X(t)=\int_{-\infty}^{\infty} h(t-\tau)\mathrm{d}W(\tau)=\int_{0}^{t} h(t-\tau)\mathrm{d}W(\tau)=\sum_{i=1}^{n}\int_{t_{i-1}}^{t_i} h(t-\tau)\mathrm{d}W(\tau)+\int_{t_n}^{t} h(t-\tau)\mathrm{d}W(\tau)$$

Approximation

Constant

Omitted

$$X(t)\approx\sum_{i=1}^{n} h(t-t_i)\int_{t_{i-1}}^{t_i}\mathrm{d}W(\tau) \qquad t_i=i\times\Delta t,\ \ i=0,1,\cdots,N \qquad t\in[t_n,t_{n+1}),\ \ n=1,2,\cdots,N$$

2 Orthogonal random variables

$$W_i=\int_{t_{i-1}}^{t_i}\mathrm{d}W(\tau)$$

$$V_i=\frac{W_i}{\sqrt{\Delta t}}$$

Satisfying

$$E[W_i]=0,\quad E[W_iW_j]=\Delta t\delta_{ij}$$

$$E[V_i]=0,\quad E[V_iV_j]=\delta_{ij}$$

★ 1. The time-domain representations for stochastic processes can be formulated as a linear combination of standard orthogonal random variables modulated by deterministic functions.

★ 2. Random variables is a set of standard orthogonal random variables whose probability distributions are not predetermined. This flexibility exactly opens possibilities for alternative representations of the orthogonal random variable set.

3 Discrete time-domain representations

A) Stationary case

$$X(t)\approx\sqrt{\Delta t}\sum_{i=1}^{n} h(t-t_i)V_i$$

B) Non-stationary case

$$\tilde{X}(t)\approx\sqrt{\Delta t}\sum_{i=1}^{n}\tilde{h}(t,t-t_i)V_i$$

standardized form

$$X^{S}(t)\approx\sigma_X\sum_{i=1}^{n}u_i(t)V_i$$

$$\tilde{X}^{S}(t)\approx\sigma_{\tilde{X}}\sum_{i=1}^{n}\tilde{u}_i(t)V_i$$

where

$$u_i(t)=\frac{h(t-t_i)}{\sqrt{\sum_{j=1}^{n}h^2(t-t_j)}}$$

$$\tilde{u}_i(t)=\frac{\tilde{h}(t,t-t_i)}{\sqrt{\sum_{j=1}^{n}\tilde{h}^2(t,t-t_j)}}$$

Convolution commutativity

$$X(t)\approx\sqrt{\Delta t}\sum_{i=0}^{n} h(t_i)V_{n-i}$$

$$\tilde{X}(t)\approx\sqrt{\Delta t}\sum_{i=0}^{n}\tilde{h}(t,t_i)V_{n-i}$$

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1. Research Background
2. Theoretical Basis
3. DR representations in time domain
4. Numerical examples
5. Conclusion

DR representations in time domain

Wuhan Institute of Technology

Construct orthogonal random variables from elementary random variables: $E\left[V_i\right]=0,\quad E\left[V_iV_j\right]=\delta_{ij}$

1 DR representations based on random orthogonal functions

a. Legendre orthogonal polynomial functions

$$\bar{V}_k=\sqrt{2k+1}P_k(\Theta),\quad k=1,2,\cdots,N$$

Elementary random variable

Uniform distribution: -1, 0, 1; $\Theta\in(-1,1)$

Legendre polynomial function

$$\begin{cases} P_{k+1}(x)=\dfrac{2k+1}{k+1}xP_k(x)-\dfrac{k}{k+1}P_{k-1}(x) \\ P_0(x)=1,\quad P_1(x)=x \end{cases}$$

b. Hartley orthogonal basis

$$\bar{V}_k=\sqrt{2}\sin(k\Theta+\alpha),\quad k=1,2,\cdots,N$$

Elementary random variable

Uniform distribution: 0, π, 2π; $\Theta\in(0,2\pi)$

Inverse transform method | **Gaussian distribution**

$$\bar{V}_k=\Phi^{-1}\left[\frac{1}{2}+\frac{1}{\pi}\arcsin\sin(k\Theta+\alpha)\right]$$

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp\left(-\frac{t^2}{2}\right)\mathrm{d}t$$

c. Trigonometric orthogonal basis

$$\bar{V}_{2k-1}=\sqrt{2}\sin(k\Theta_1+\alpha),\quad \bar{V}_{2k}=\sqrt{2}\cos(k\Theta_2+\alpha),\quad k=1,2,\cdots,N/2$$

Elementary random variable $\quad\Theta_1,\Theta_2\in(0,2\pi)$

Deterministic one-to-one mapping relationship $\quad\left\{\bar{V}_k\right\}_{k=1}^{N}\rightarrow\left\{V_i\right\}_{i=1}^{N}$

2 Implementation scheme

Start

↓

Select the representative point sets of elementary random variables by equidistant partition or the NTM

↓

Calculate the deterministic values of the standard orthogonal random variable sets $\left\{\bar{V}_k\right\}_{k=1}^{N}$

↓

Generate the representative values of the standard orthogonal random variable sets $\left\{V_i\right\}_{i=1}^{N}$

↓

Generate the representative sample sets of both stationary and non-stationary stochastic processes by time-domain representations

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Numerical examples

◆ Simulation of stationary ground motion processes

Calculation parameters for simulation of stationary ground motions

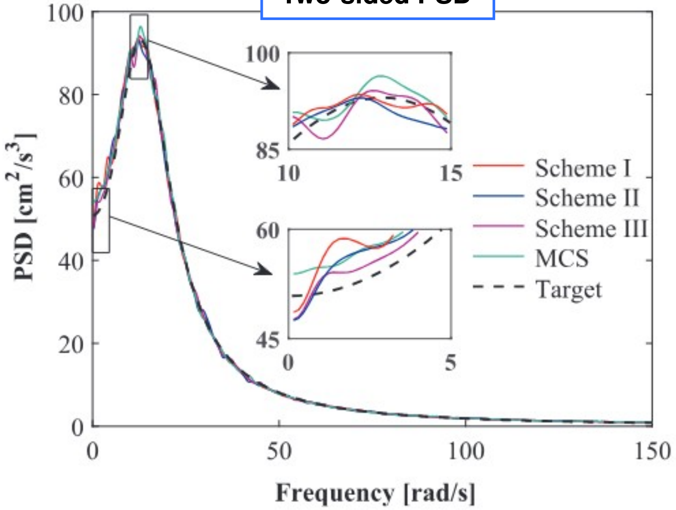
Parameters	Values	Parameters	Values
Dominant frequency	ω_g =5π rad/s	Damping ratio	ξ_g =0.6
PGA	A_{max} =200 cm/s^2	Peak factor	γ=2.8
The entire duration	T = 30 s	Time step	Δt =0.01 s
The upper cut-off frequency	ω_u =100π rad/s	Frequency interval	$\Delta\omega$ =0.209 rad/s

The parameters are based on the values for stiff soils.

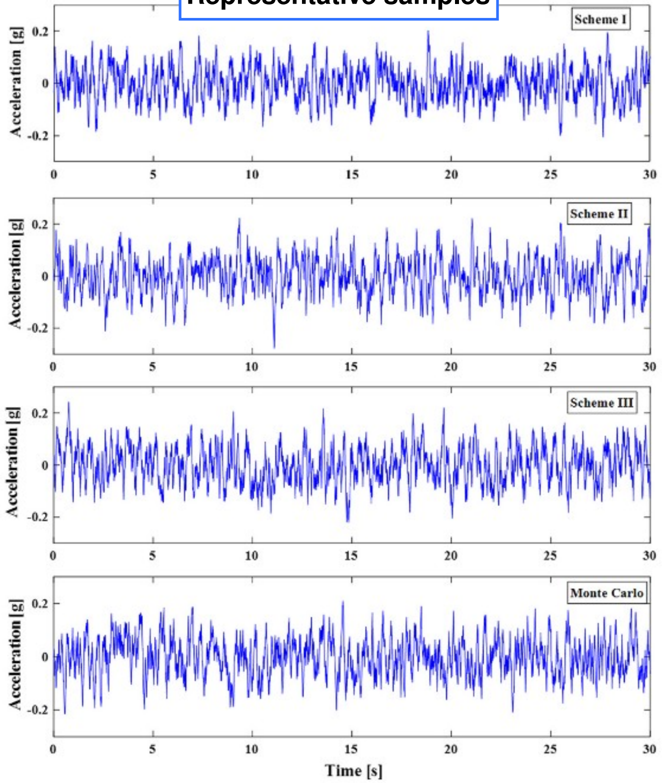
Comparison of simulation accuracy and efficiency between DR and MCS

Simulation method	Scheme	M-ARE (%)	SD-ARE (%)	PSD-ARE (%)	Time (s)
DR(233)	I	3.98×10^{-3}	3.67	2.65	3.87
	II	3.73×10^{-4}	2.57	2.46	3.89
	III	9.60×10^{-11}	1.39	2.49	4.41
MCS(233)		5.22	3.51	2.76	4.40
MCS(500)		3.74	2.44	2.50	8.84

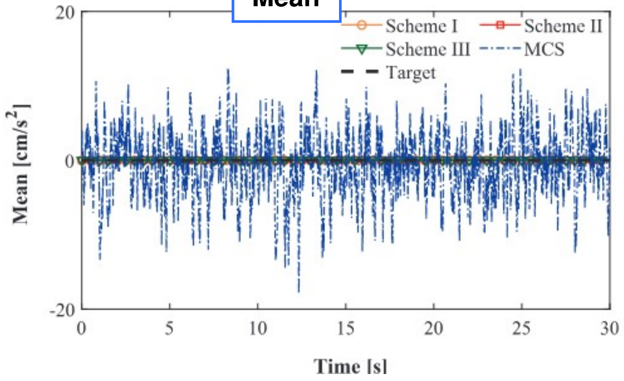
Two-sided PSD



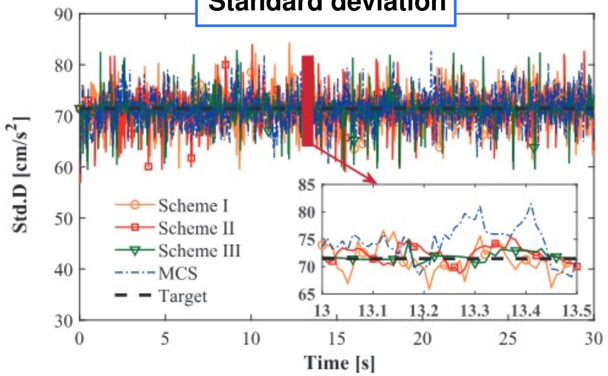
Representative samples



Mean



Standard deviation



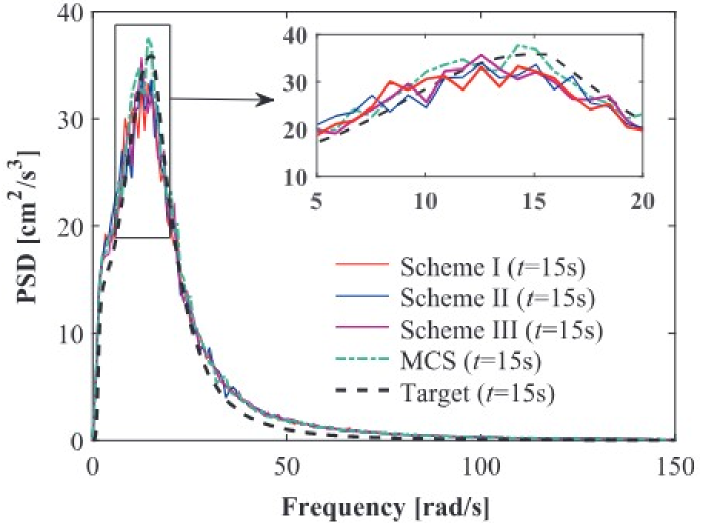
Numerical examples

◆ Simulation of non-stationary ground motion processes

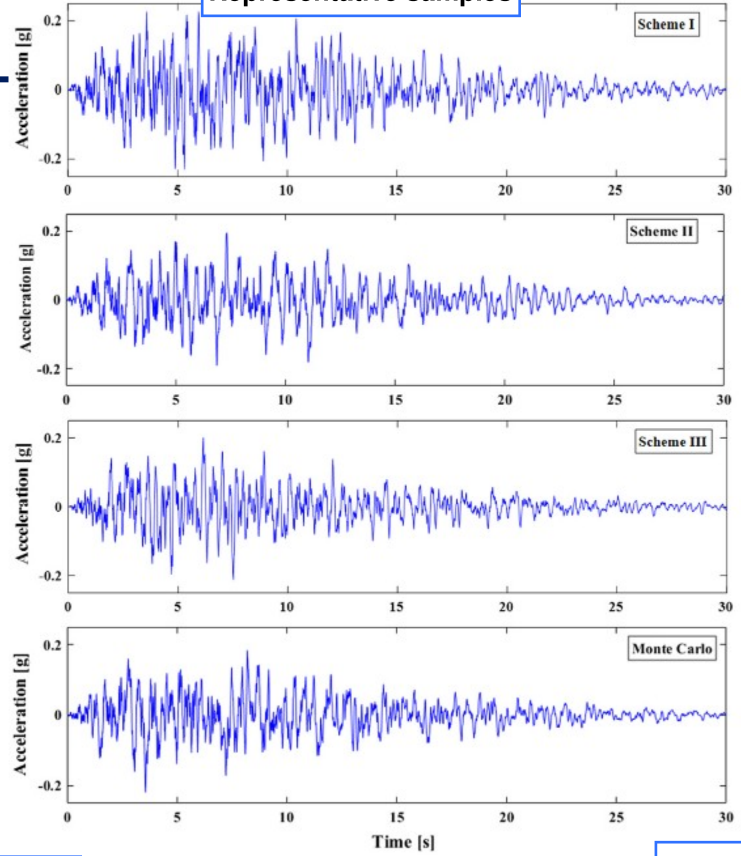
Calculation parameters for simulation of non-stationary ground motions

Parameters	Values
Dominant frequency of the rock bed	ω_{f} =0.5π rad/s
Damping ratio of the rock bed	ξ_{f} =0.6
Parameter of the time-frequency modulation function	a =0.15 s^{-1}
Number of representative samples	n_{sel} =233

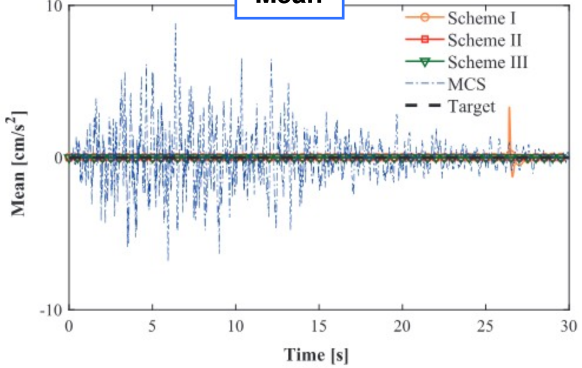
PSD at the time instant of t=15s



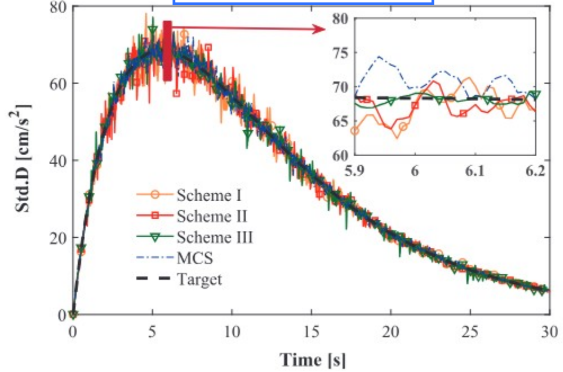
Representative samples



Mean



Standard deviation



Numerical examples

◆ Simulation of non-stationary ground motion processes

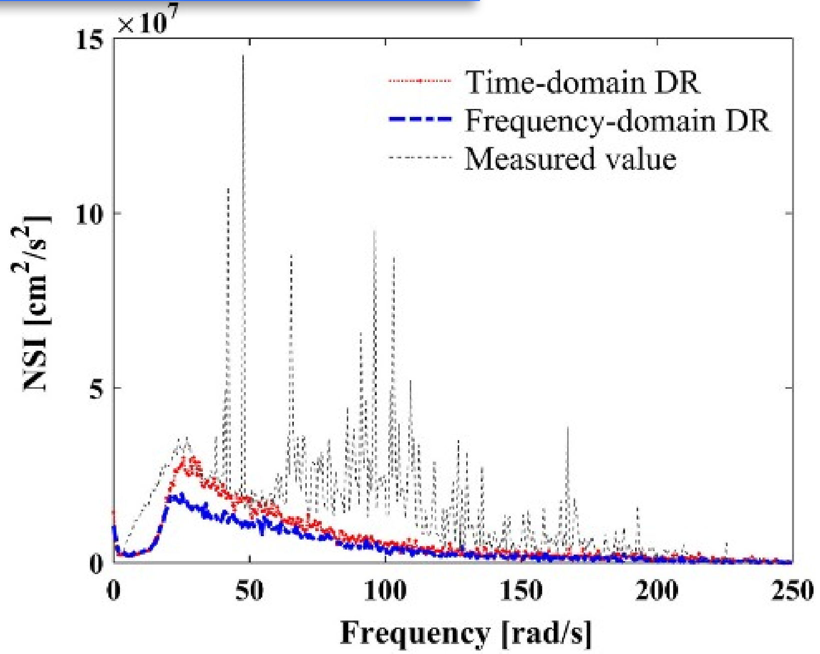
Non-stationarity index (NSI)

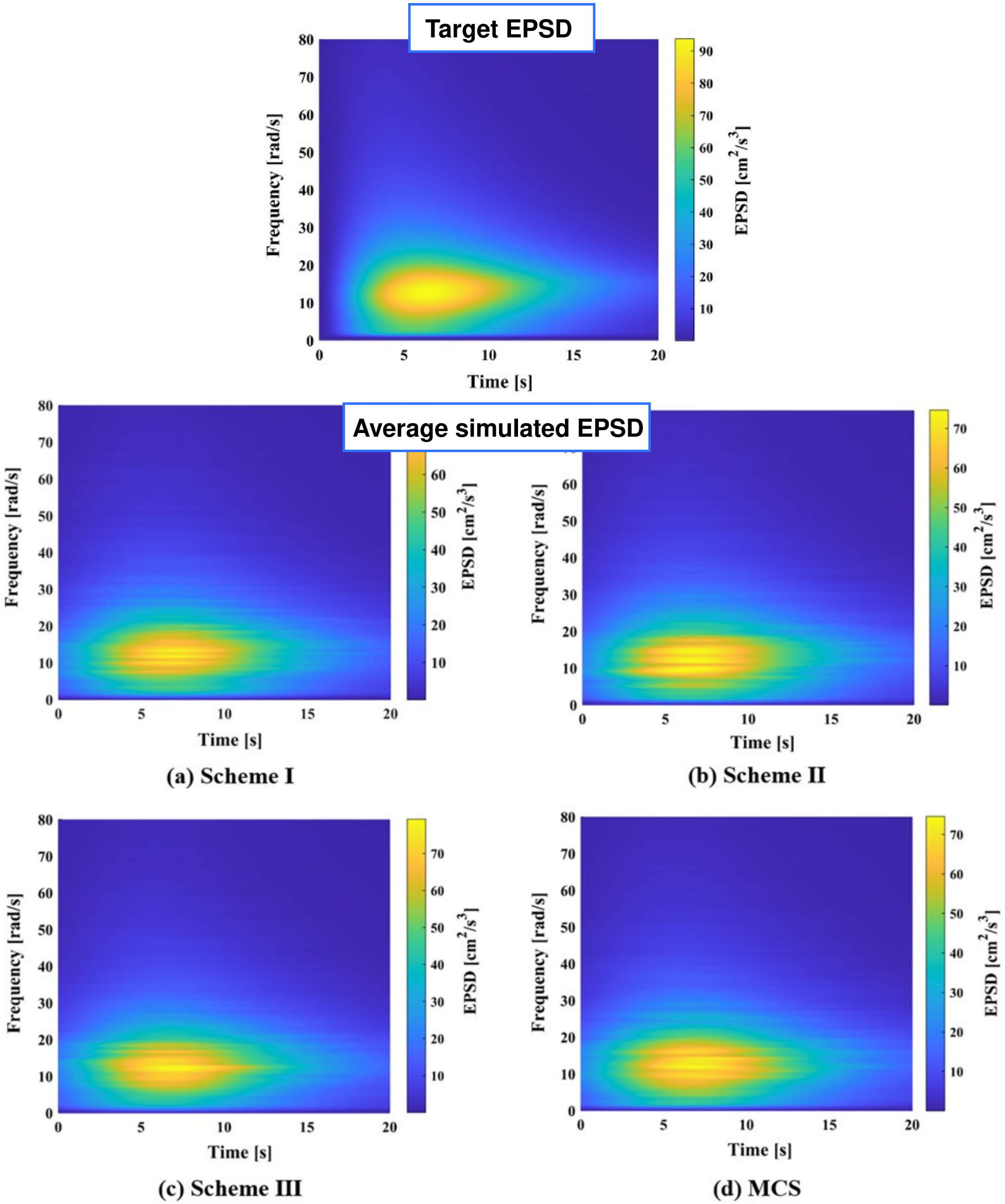
$$\mathrm{NSI}(\omega) = \frac{1}{N}\sum_{i=0}^{N-1}\left(1 - \frac{\Psi_{\bar{X}}(\omega, t_i)}{\psi_{\bar{X}}(\omega)}\right)^2$$

Hilbert spectrum

Boundary spectrum

523 sets of sample records for stiff soils





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Conclusion

1 Unified theoretical form

Through the derivation of the time-domain representations for stochastic processes, two equivalent representations of the stochastic processes in both frequency and time domains are clarified. Essentially, regardless of whether it is in the frequency or time domain, both stationary and non-stationary stochastic processes can be discretized into a linear superposition form of a set of mutually uncorrelated random variables modulated by deterministic functions.

2 Accuracy and efficiency

Compared with the conventional MCS, the DR simulation method offers satisfactory accuracy. At the meantime, the proposed trigonometric orthogonal basis achieves the comparable accuracy of MCS with a significantly reduced samples size, which not only reduces the time needed to generate stochastic process samples but also enhances the efficiency of nonlinear stochastic dynamic analysis for complex engineering structures.

3 Non-stationary characteristics

The NSI of the representative ground motion samples generated by the time-domain DR method more closely resembles that of real earthquake scenarios compared to the frequency-domain DR method, demonstrating that the time-domain DR method exhibits superior capability in capturing non-stationary characteristics.

4 Complete probabilistic set

By means of the deterministic point-selection methods for elementary random variables, all representative samples with assigned probabilities can thus constitute a complete probability set. As a result, the DR method provides a solid foundation to combine the PDEM for the accurate nonlinear stochastic dynamic response analysis and refined reliability assessment of complex engineering structures at full probability level.

New Book Introduction: English Monograph

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New Book Introduction: English Monograph

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Thanks for watching

Questions are invited

Speaker: Xiaojiao Fu, Zhangjun Liu
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