TIME-DOMAIN DIMENSION-REDUCTION SIMULATION FOR NON-STATIONARY STOCHASTIC GROUND MOTION

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Abstract

Accurate simulation of stochastic ground motions is crucial for structural reliability assessment. Stochastic ground motion simulation methods are mainly classified into frequency-domain and time-domain methods. Among them, the time-domain method enables direct simulation of stochastic processes without frequency decomposition. However, in time-domain methods, the conventional Monte Carlo (MC) method is restricted by the curse of dimensionality due to the need to introduce thousands of random variables, thus making dimension-reduction (DR) methods a research priority. The core idea of DR methods is to construct random orthogonal functions, including Legendre polynomials, Gaussian Hartley orthogonal basis, and Trigonometric orthogonal basis, and take them as constraint conditions for orthogonal random variables, thereby reducing the number of random variables required for simulation from thousands to only one or two. As such, the time-domain dimension-reduction representation can be regarded as the linear superposition of deterministic functions modulated by a series of orthonormal random variables. This paper conducts the simulation of non-stationary stochastic ground motions based on the time-domain dimension-reduction representation. Results of numerical examples show that compared with the conventional MC method, the proposed time-domain DR method achieves both higher calculation accuracy and efficiency. Meanwhile, in comparison with the frequency-domain DR method, this method has more prominent advantages in characterizing the non-stationary characteristics of stochastic ground motions. The above analysis results show that the non-stationary stochastic ground motion samples generated by the DR method have definite probabilistic attributes and form a complete probabilistic sample set, which can be combined with advanced theories such as the probability density evolution method (PDEM) to conduct refined stochastic dynamic response analysis and reliability assessment of complex engineering structures.

Keywords*: non-stationary stochastic ground motions; time-domain method; dimension-reduction representation; random orthogonal functions*

1. Introduction

Earthquake ground motions exhibit significant randomness in both time and space, posing critical challenges to structural safety. Therefore, treating such excitations as non-stationary stochastic processes and simulation of such processes has become indispensable for seismic analysis and reliability assessment of engineering structures [1]. In this context, existing approaches mainly include frequency-domain methods (e.g., SRM and POD) and time-domain methods (e.g., the filtered white noise process model and ARMA) [2, 3]. Among them, frequency-domain methods are efficient for stationary processes but have limitations in capturing the time–frequency non-stationarity of seismic ground motions. In contrast, time-domain methods provide a more direct description of correlation structures and are more suitable for simulating non-stationary stochastic processes [4]. Notably, most of the above methods are developed within the framework of conventional Monte Carlo simulation (MCS), which typically requires a large number of random variables and samples, resulting in high computational cost and reduced accuracy. To address this issue, this study introduces the dimension-reduction (DR) approach based on random orthogonal functions [5] within the filtered white noise process model, proposes a time-domain dimension-reduction simulation method for non-stationary stochastic ground motion. The proposed method enables an efficient representation of non-stationary processes using only one or two random variables. Moreover, the generated representative samples form a complete probability set, which can be naturally integrated with the probability density evolution method (PDEM) to enable refined stochastic dynamic response analysis and reliability assessment of complex engineering structures.

2. Time-domain representations for non-stochastic processes

According to Priestley's spectral decomposition theory [6], a zero-mean real-valued non-stationary stochastic process $\tilde{X}(t)$ can be represented by the following Fourier–Stieltjes integral in the frequency domain:

$$\tilde{X}(t)=\int_{-\infty}^{\infty}A(t,\omega)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}Z(\omega)=\int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}\omega t}A(t,\omega)H(\omega)\mathrm{d}U(\omega) \tag{1}$$

where i denotes the imaginary unit. $A(t,\omega)$ denotes the deterministic real-valued time–frequency modulation function. $H(\omega)$ is the frequency response function satisfying $H(-\omega)=H^*(-\omega)$. $Z(\omega)$ and $U(\omega)$ are orthogonal increment processes, whose increments satisfy the following basic conditions:

$$E\left[\mathrm{d}Z(\omega)\right]=E\left[\mathrm{d}U(\omega)\right]=0,\ \mathrm{d}Z(-\omega)=\mathrm{d}Z^*(\omega),\ \ \mathrm{d}U(-\omega)=\mathrm{d}U^*(\omega) \tag{2a}$$

$$E\left[\mathrm{d}Z(\omega)\mathrm{d}Z^*(\omega')\right]=\frac{1}{2\pi}S_X(\omega)\delta_{\omega\omega'}\mathrm{d}\omega,\ E\left[\mathrm{d}U(\omega)\mathrm{d}U^*(\omega')\right]=\frac{1}{2\pi}\delta_{\omega\omega'}\mathrm{d}\omega \tag{2b}$$

where $E[\cdot]$ and '$*$' denote the mathematical expectation and complex conjugate, respectively. $\delta_{\omega\omega'}$ is the Kronecker-delta, i.e., $\delta_{\omega\omega'}=1$ if $\omega=\omega'$, or $\delta_{\omega\omega'}=0$ if $\omega\neq\omega'$. $S_X(\omega)$ is the two-sided power spectral density (PSD) function of the stationary stochastic process, satisfying $S_X(\omega)=|H(\omega)|^2$.

For the non-stationary stochastic process $\tilde{X}(t)$, the evolutionary PSD (EPSD) function satisfying $S_{\tilde{X}}(t,\omega)=|A(t,\omega)|^2 S_X(\omega)$ [6].Now, let $\tilde{H}(t,\omega)=A(t,\omega)H(\omega)$, $S_{\tilde{X}}(t,\omega)$ can be further defined as follows:

$$S_{\tilde{X}}(t,\omega)=\left|\tilde{H}(t,\omega)\right|^2 \tag{3}$$

For each given time instant t, the inverse Fourier transform of $\tilde{H}(t,\omega)$ can be defined as the function $\tilde{h}(t,\tau)$. The following Fourier transform pairs can be constructed:

$$\begin{cases} \tilde{H}(t,\omega) = \int_{-\infty}^{\infty} \tilde{h}(t,\tau) \mathrm{e}^{-\mathrm{i}\omega\tau} \mathrm{d}\tau \\ \tilde{h}(t,\tau) = \int_{-\infty}^{\infty} \tilde{H}(t,\omega) \mathrm{e}^{\mathrm{i}\omega\tau} \mathrm{d}\omega \end{cases} \tag{4}$$

Note that $\tilde{h}(t,\tau)$ is a real-valued function.

To this end, by substituting Eq. (4) into Eq. (1) and exchanging the order of double integrals, Eq. (1) can be rewritten as follows:

$$\tilde{X}(t) = \int_{-\infty}^{\infty} \tilde{h}(t,t-\tau) \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega\tau} \mathrm{d}U(\omega) \mathrm{d}\tau = \int_{-\infty}^{\infty} \tilde{h}(t,t-\tau) \mathrm{d}W(\tau) \tag{5}$$

where $W(\tau)$ can be regarded as a white noise process with $\mathrm{d}W(\tau) = w(\tau)\mathrm{d}\tau$ [7]. Both $\mathrm{d}W(\tau)$ and $w(\tau)$ satisfies the following basic conditions:

$$E\left[\mathrm{d}W(\tau)\right] = 0, \ E\left[\mathrm{d}W(\tau)\mathrm{d}W^*(\tau')\right] = \delta_{\tau\tau'}\mathrm{d}\tau \tag{6a}$$

$$E\left[w(\tau)\right] = 0, \ E\left[w(\tau)w^*(\tau')\right] = \delta(\tau-\tau') \tag{6b}$$

where $w(\tau)$ denotes a zero-mean stationary white noise process with the two-sided PSD $S_w(\omega)=1$. $\delta(\cdot)$ denotes the Dirac delta function.

For the convenience of simulation, Eq. (5) is typically rewritten as a piecewise integral form with practical significance as follows:

$$\tilde{X}(t) = \int_{-\infty}^{\infty} \tilde{h}(t,t-\tau)\mathrm{d}W(\tau) = \int_{-\infty}^{t} \tilde{h}(t,t-\tau)\mathrm{d}W(\tau) = \sum_{i=1}^{n} \int_{t_{i-1}}^{t_i} \tilde{h}(t,t-\tau)\mathrm{d}W(\tau) + \int_{t_n}^{t} \tilde{h}(t,t-\tau)\mathrm{d}W(\tau) \tag{7}$$

where $t_i = i \times \Delta t$, $i = 0,1,\cdots,N$; Δt denotes the time interval; N indicates the number of discrete terms in the entire duration T; n signifies the number of discrete terms within the current time t.

Now, assuming $\tilde{h}(t,t-\tau)$ remains constant over a short time interval $t_{i-1} \le \tau \le t_i$ and dropping the last term in Eq. (7), the equation is simplified to:

$$\tilde{X}(t) \approx \sum_{i=1}^{n} \tilde{h}(t,t-t_i) \int_{t_n}^{t_i} \mathrm{d}W(\tau) = \sum_{i=1}^{n} \tilde{h}(t,t-t_i) W_i, \ t \in \left[t_n, t_{n+1}\right), \ n = 1,2,\cdots,N \tag{8}$$

where W_i $(i=1,2,\cdots N)$ is a set of zero-mean uncorrelated random variables, satisfying the following conditions:

$$E\left[W_i\right] = 0, \ E\left[W_i W_j\right] = \Delta t \delta_{ij} \tag{9}$$

Through standardization (namely $V_i = W_i/\Delta t$), the time-domain representation of the non-stationary stochastic processes can be rewritten as follows:

$$\tilde{X}(t) \approx \sqrt{\Delta t} \sum_{i=1}^{n} \tilde{h}(t,t-t_i) V_i \tag{10}$$

where V_i $(i=1,2,\cdots N)$ is a set of standard orthogonal random variables, satisfying the following conditions:

$$E\left[V_i\right]=0\,,\ E\left[V_iV_j\right]=\delta_{ij} \tag{11}$$

3. DR representations of the non-stationary stochastic processes in time domain

To achieve time-domain dimension-reduction simulation of non-stationary stochastic ground motion, the orthogonal random variable sets $\left\{\bar{V}_k\right\}_{k=1}^{N}$ can be conveniently constructed in the form of random orthogonal functions following the dimension-reduction concept [5]. and is defined as follows:

$$\bar{V}_{2k-1}=\sqrt{2}\sin(k\Theta_1+\alpha)\,,\ \bar{V}_{2k}=\sqrt{2}\cos(k\Theta_2+\alpha)\,,\ k=1,2,\cdots,N/2 \tag{12}$$

where Θ_1 and Θ_2 indicate mutually independent elementary random variables with uniform distribution over the interval $(0,2\pi)$. The parameter α is valued by $\alpha=\pi/4$.

Note that the predefined standard orthogonal random variable sets $\left\{\bar{V}_k\right\}_{k=1}^{N}$ need to be further transformed to the target standard orthogonal random variable sets $\left\{V_i\right\}_{i=1}^{N}$ in Eq. (10) through a deterministic one-to-one mapping relationship, i.e., $\left\{\bar{V}_k\right\}_{k=1}^{N}\rightarrow\left\{V_i\right\}_{i=1}^{N}$. This deterministic transformation relationship can be realized using the built-in functions of the MATLAB toolbox, namely rand (‘state’,0) and temp = randperm (*N*). This precisely constitutes a sufficient condition of the DR method for simulating non-stationary stochastic processes.

4. Numerical example

In order to accurately capture the non-stationary characteristics of ground motion process $\tilde{X}(t)$, this study employs the following time-frequency modulation function[8]:

$$A(t,\omega)=\frac{\exp(-at)-\exp[-(c\times\left|\omega-\omega_{\mathrm{g}}\right|+b)\times t]}{\exp(-at^*)-\exp[-(c\times\left|\omega-\omega_{\mathrm{g}}\right|+b)\times t^*]},\quad t>0,\quad \omega>0 \tag{13a}$$

where

$$t^*=\frac{\ln(c\times\left|\omega-\omega_{\mathrm{g}}\right|+b)-\ln(a)}{c\times\left|\omega-\omega_{\mathrm{g}}\right|+(b-a)},\quad \omega>0 \tag{13b}$$

where ω_{g} denote the dominant frequency of the soil layer. The parameters $b=a+0.001$ and $c=0.005$.

For the two-side PSD function of stationary ground motion processes, this paper adopts the widely-used Clough-Penzien model [9]:

$$S_X(\omega)=\frac{\omega_{\mathrm{g}}^4+4\xi_{\mathrm{g}}^2\omega_{\mathrm{g}}^2\omega^2}{(\omega^2-\omega_{\mathrm{g}}^2)^2+4\xi_{\mathrm{g}}^2\omega_{\mathrm{g}}^2\omega^2}\cdot\frac{\omega^4}{(\omega^2-\omega_{\mathrm{f}}^2)^2+4\xi_{\mathrm{f}}^2\omega_{\mathrm{f}}^2\omega^2}S_0 \tag{14}$$

where ξ_{g} denote the critical damping of the soil layer. S_0 demonstrates the spectral intensity factor, defined as follows:

$$S_0 = \frac{2A_{\max}^2}{\gamma^2 \omega_g \left(2\xi_g + \frac{1}{2\xi_g}\right)} \tag{15}$$

where $A_{\max}$ denotes the peak ground acceleration (PGA) value. γ refers to the peak factor. In addition, the specific model parameters in the numerical example are shown in Table 1.

Table 1 – Calculation parameters for simulation

Parameters	**Values**
Dominant frequency	$\omega_g = 5\pi$ rad/s
PGA	$A_{\max} = 200$ cm/s^2
The entire duration	$T = 30$ s
The upper cut-off frequency	$\omega_u = 100\pi$ rad/s
Damping ratio	$\xi_g = 0.6$
Peak factor	$\gamma = 2.8$
Time step	$\Delta t = 0.01$ s
Frequency interval	$\Delta\omega = 0.209$ rad/s

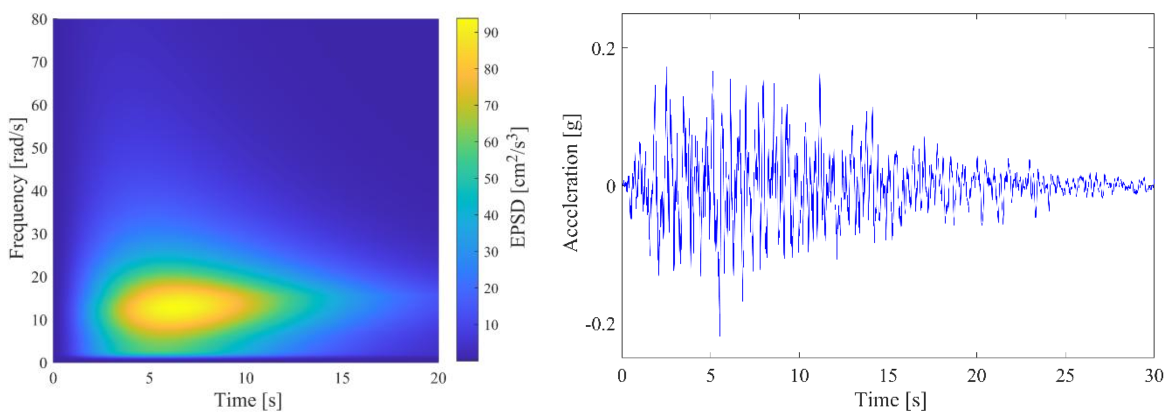

Fig. 1 – Target EPSD and the 100th representative samples of non-stationary process

Fig.1 presents the target EPSD function when $\omega \geq 0$ and the 100th representative samples of non-stationary seismic acceleration time-histories generated by the proposed method. It can be seen that the seismic acceleration mainly concentrates its energy in the time range of $[0,15]$s and the frequency range of $[0,30]$ rad/s . As observed, the generated representative samples clearly exhibit non-stationary characteristics.

Table 2 – Comparison of simulation accuracy and efficiency between DR and MCS

Simulation method	**M-ARE (%)**	**SD-ARE (%)**	**Time (s)**
DR	9.60×10^{-11}	1.39	10.07
MCS	5.22	3.51	3.51

Table 2 compares the performance of the proposed DR method and the conventional MC in terms of computational accuracy and efficiency. The results show that the DR method offers significantly higher accuracy than the MCS method while maintaining an acceptable computational time, demonstrating its effectiveness.

Fig.2 compares the second-order statistics (mean and standard deviation) of the generated time-histories with the target values. Even with only 233 representative time-histories, the proposed time-domain DR method yields results that closely match the targets, confirming its effectiveness.

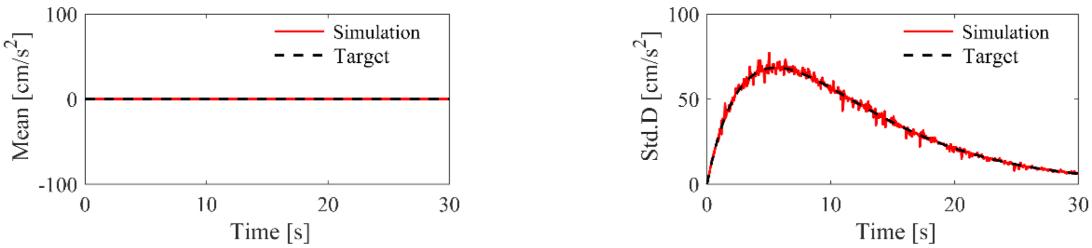

Fig. 2 –Mean and standard deviation of seismic acceleration process (Simulated vs. target)

Fig.3 shows the average EPSD function and the PSD comparison at t=15s for processes simulated via the proposed method. Both the energy distribution of the time-domain DR simulation and the simulated EPSD demonstrate high fidelity to the target spectrum, validating the feasibility of the proposed approach.

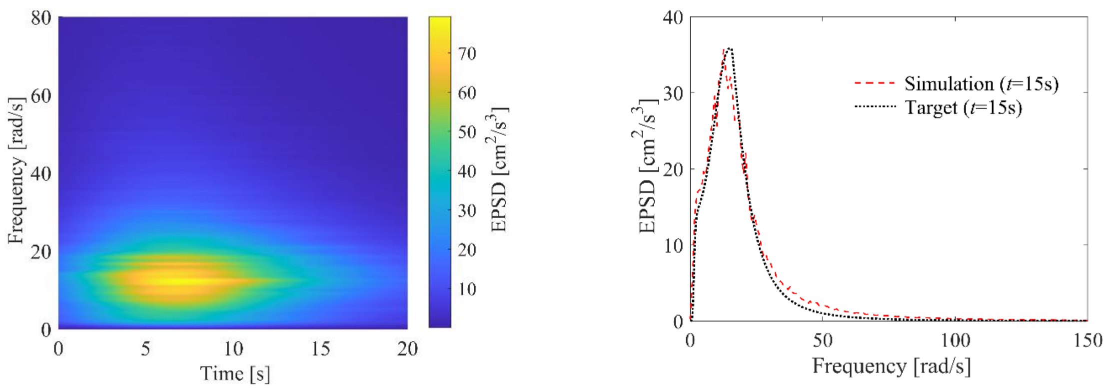

Fig. 3 – Average simulated EPSD function and EPSD at t=15s (simulated vs. target) for non-stationary seismic acceleration processes

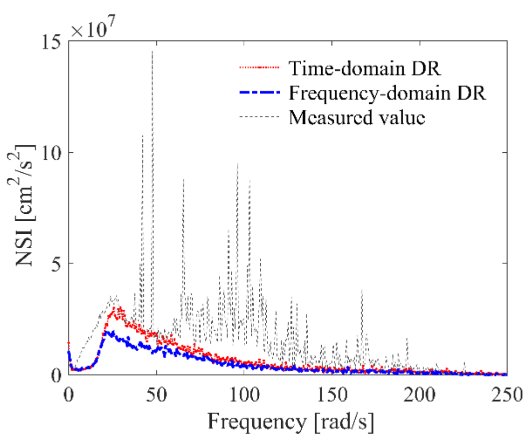

Fig. 4 – NSI comparison with measured data: Time-domain vs. Frequency-domain DR

The non-stationarity index (NSI) of a signal can be computed using localized wavelet decomposition and Hilbert transform [10]. Accordingly, Fig. 4 compares the NSI variations between average of samples generated by time-domain and frequency-domain DR methods and measured records. The time-domain method is based on a filtered white noise model, while the frequency-domain method uses spectral representation. Under identical conditions, the time-domain results show better agreement with recorded data in non-stationary characteristics, confirming the superiority of the time-domain framework.

5. Conclusion

A time-domain DR representation for simulating stationary and non-stationary stochastic processes is developed within a linear filter framework. By introducing random orthogonal functions, the method significantly reduces the number of required random variables while maintaining accuracy comparable to MCS. The formulation unifies time-domain and frequency-domain representations. Notably, the time-domain DR method yields NSI values closer to real ground motions than the frequency-domain approach, indicating its superior capability in capturing non-stationary characteristics. Moreover, the generated samples preserve complete probabilistic properties, enabling efficient stochastic dynamic analysis and reliability assessment of complex engineering structures.

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